ECON 4910 Envvironmental economics; spring 2014

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Lecture note 7: Climate policy I (taxes, quotas)

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Please bring lecture note and EEAG report to lecture.

Reading:
Perman et al. (2011). Sections 9.5 and 16.1
EEAG (2012)

Important features of the climate issue:

- stock pollution
- potentially large damage; partial equilibrium analysis may be misleading (see below and Perman 16.1)
- large uncertainties
- distributional issues
- CO_2 from burning exhaustible fossil fuels (Lecture 10)
- international dimension (Lecture 9)

A simple IAM

Time references are omitted where this cannot cause any misunderstanding. Maximize

$$\int_0^\infty e^{-\rho t} W(\underset{+}{C}, \underset{-}{S}) dt$$

s.t.

$$\dot{K} = F(K, x, t) - C$$
$$\dot{S} = x - \delta S$$

To simplify the analysis we assume

$$W(C, S) = u(C) - hS$$

Notice that the marginal cost (WTP) of S is hC^{α} under the assumption that $u(C) = \frac{1}{1-\alpha} C^{1-\alpha}$, i.e. $u'(C) = C^{-\alpha}$.

The current value Hamiltonian is

$$H = u(C) - hS + \lambda \left[F(K, x, t) - C \right] + \mu \left[x - \delta S \right]$$

and the optimum conditions are

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial K} = (\rho - F_K)\lambda \qquad (1)$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial S} = (\rho + \delta)\mu + h \qquad (2)$$

$$H_C = u'(C) - \lambda = 0 \qquad (3)$$

$$H_x = \lambda F_x + \mu = 0 \qquad (4)$$

Using $u' = C^{-\alpha}$ and $g = \frac{\dot{C}}{C}$, equations (1) and (3) give

$$F_K = \rho + \alpha g \equiv r \tag{5}$$

Rewriting (4) gives

$$F_x = \frac{-\mu}{\lambda} \equiv q \tag{6}$$

which together with (2) implies

$$\frac{\dot{q}}{q} = \frac{\dot{\mu}}{\mu} - \frac{\dot{\lambda}}{\lambda} = \left[\rho + \delta + \frac{h}{\mu}\right] - [\rho - r]$$

Using (6) this may be rewritten as

$$\frac{\dot{q}}{q} = r + \delta - \frac{h}{qu'}$$

or

$$\dot{q} = (r + \delta)q - hC^{\alpha} \tag{7}$$

If we as a simplification regard r as constant, this gives (using the transversality condition of the optimization problem)

$$q(t) = \int_t^{\infty} e^{-(r+\delta)(\tau - t)} hC(t)^{\alpha} e^{\alpha g((\tau - t))} d\tau$$

or, using (5)

$$q(t) = hC(t)^{\alpha} \int_t^{\infty} e^{-(\rho+\delta)(\tau - t)} d\tau = \frac{h}{\rho + \delta} C(t)^{\alpha}$$

or, if $C(t) = C_0 e^{gt}$

$$q(t) = \frac{hC_0}{\rho + \delta} e^{\alpha g t} \tag{8}$$

which is incrasing over time. Using (6) we also see that

$$e^{-rt} q(t) = \frac{hC_0}{\rho + \delta} e^{-\rho t} \tag{9}$$

which is declining over time.